

المترين المتكافئ

$Ax = B$  المتكافئ مع

$$\begin{pmatrix} 2 & 1 & -1 \\ 4 & 3 & 1 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix} \quad (1)$$

مصفوفة المصفوفة

$$(A|B) = \left( \begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 4 & 3 & 1 & 7 \\ -2 & 1 & 2 & 2 \end{array} \right)$$

3 خطوات حروف حروف

$$(A^{(1)}|B^{(1)}) = \left( \begin{array}{ccc|c} 1 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 3 & 5 \\ 0 & 2 & 1 & 3 \end{array} \right)$$

2 خطوات

$$A^{(2)}|B^{(2)} = \left( \begin{array}{ccc|c} 1 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \end{array} \right)$$

3 خطوات

$$(A^{(3)}|B^{(3)}) = \left( \begin{array}{ccc|c} 1 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 7/5 \end{array} \right)$$

4 خطوات

$Ax = B \Rightarrow A^{(3)}x = B^{(3)}$

$$\begin{pmatrix} 1 & 1/2 & -1/2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/2 \\ 5 \\ 7/5 \end{pmatrix}$$

$z = 7/5 \Rightarrow y + 3z = 5$

$y = 5 - 3(7/5) = 4/5$

المترين المتكافئ

$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$  (1)

$B = A^2 A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$  (2)

$B = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$  (3)

$\det(B - \lambda I_2) = 0$

$$B - \lambda I_2 = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 1-\lambda & 2 \\ 2 & 5-\lambda \end{pmatrix}$$

$\det(B - \lambda I_2) = 0$

$\Rightarrow (1-\lambda)(5-\lambda) - 4 = 0$

$\Rightarrow \lambda^2 - 6\lambda + 1 = 0$

$\Delta = b^2 - 4ac$

$= (36) - 4(1)(1) = 32$

$\lambda_1 = \frac{6 - 4\sqrt{2}}{2} = 3 - 2\sqrt{2}$

$\lambda_2 = \frac{6 + 4\sqrt{2}}{2} = 3 + 2\sqrt{2}$

3(B) مساحة القطر

$f(B) = \max_{1 \leq i \leq 2} |d_i| = (3 + 2\sqrt{2})$

$\max(3 - 2\sqrt{2}, 3 + 2\sqrt{2})$

$f(B) = 3 + 2\sqrt{2}$

$\|A\|_2 = \sqrt{\rho(A^T A)}$

$= \sqrt{3 + 2\sqrt{2}}$

$\Rightarrow x + \frac{1}{2}y - \frac{1}{2}z = \frac{1}{2}$

$\Pi_1 \quad x = \frac{1}{2} + \frac{2}{10} + \frac{7}{10} = \frac{11}{10}$

درام مثال تکرار با همون مسائل

$$\begin{cases} x = \frac{1}{3} + \frac{1}{3}y + \frac{1}{3}z \\ y = \frac{1}{5} + \frac{1}{5}x - \frac{1}{5}z \\ z = \frac{1}{6} - \frac{1}{6}x + \frac{1}{6}y \end{cases}$$

مع معروضه تکرار با همون مسائل

$$\begin{cases} x = -\frac{1}{4}y - \frac{1}{4}z + \frac{7}{4} \\ y = -\frac{1}{5}z + \frac{5}{4} \\ z = \frac{1}{6} \end{cases}$$

$$\begin{cases} x = -\frac{1}{4}y - \frac{1}{4}z + \frac{7}{4} \\ y = -\frac{1}{5}z + \frac{5}{4} \\ z = \frac{1}{6} \end{cases}$$

$$B_{\text{Bas}} = \begin{pmatrix} 0 & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & -\frac{1}{5} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\|B_{\text{Bas}}\|_{\infty} = \max(0, \frac{1}{4}, \frac{1}{5}, 0) = 0,25$$

$$\max(|\frac{2}{4}|, |\frac{1}{5}|, 0) = 0,5$$

درام مثال تکرار با همون مسائل

$$\|B_{\text{Bas}}\|_{\infty} = 0,5 < 1$$

تمرین شماره ۳

۱. معادله فشرده  
 $A = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 6 \end{pmatrix}$

۲.  $\sum_{j=1}^3 |a_{ij}| > |a_{ii}|$  (1)

$4 = |a_{11}| = |4| > |a_{12}| + |a_{13}| = 1 + 1 = 2$

$5 = |a_{22}| = |5| > |a_{21}| + |a_{23}| = 1 + 1 = 2$

$6 = |a_{33}| = |6| > |a_{31}| + |a_{32}| = 1 + 1 = 2$

۳. مثال تکرار با همون مسائل

$$\begin{cases} x = \frac{1}{4}y - \frac{1}{4}z + \frac{7}{4} \\ y = -\frac{1}{5}x - \frac{1}{5}z + \frac{5}{4} \\ z = -\frac{1}{6}x - \frac{1}{6}y + \frac{9}{6} \end{cases}$$

۴. معروضه تکرار با همون مسائل

$$B_j = \begin{pmatrix} 0 & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{5} & 0 & -\frac{1}{5} \\ -\frac{1}{6} & -\frac{1}{6} & 0 \end{pmatrix}$$

۵.  $B_j \rightarrow C$

$\|B_j\|_1 = \max(\frac{1}{5} + \frac{1}{6}, \frac{1}{4} + \frac{1}{6}, \frac{1}{4} + \frac{1}{5})$

$\rightarrow \max(\frac{11}{30}, \frac{10}{24}, \frac{9}{25}) = \frac{9}{25}$

$\|B_j\|_1 = 0,36 < 1$  (2)

۶. درام مثال تکرار با همون مسائل

$x = \frac{7}{4}, y = \frac{1}{5}, z = \frac{9}{6}$  (3)

(4)